THE INTEGERS

The most common numbers are those used for counting, namely the numbers 1, 2, 3, 4, ,which are called the positive integers. Even for counting, we need at leastone other number, namely,0 (zero).For instance, we may wish to count the number of right answers you mayget on a test for this course, out of a possible 100. If you get 100, then allyour answers were correct. If you get 0, then no answer was correct.The positive integers and zero can be represented geometrically on a line,in a manner similar to a ruler or a measuring stick

For this we first have to select a unit of distance, say the inch, and then onthe line we mark off the inches to the right as in the pictureFor convenience, it is useful to have a name for the positive integerstogether with zero, and we shall call these the natural numbers. Thus 0is a natural number, so is 2, and so is 124,521. The natural numbers can beused to measure distances, as with the ruler.By definition, the point represented by 0 is called the origin.The natural numbers can also be used to measure other things. For example, a thermometer is like a ruler which measures temperature. However

NUMBERS

Thus a sum involving three terms may be written in many ways, as follows:

(a - b) + c = (a + (- 6)) + c

= a + (—6 + c)

= a + (c — b)

= (a + c) — b

and we can also write this sum as

a — b + c = a + c — b,

omitting the parentheses. Generally, in taking the sum of integers, we cantake the sum in any order by applying associativity and ommutativity repeatedly.

As a special case of N3, for any integer a we have

by associativity

by commutativity

by associativity,

N4. a = —(—a).

This is true because

a + (- a) = 0,

and we can apply N3 with b = —a. Remark that this formula is truewhether a is positive, negative, or 0. If a is positive, then —a is negative.If a is negative, then —a is positive. In the geometric representation of numbers on the line, a and —a occur symmetrically on he line on opposite sides of 0. Of course, we can pile up minus signs and get other relationships,like

$- 3 = - (- (- 3))$,

or

$3 = - (- 3) = - (- (- (- 3)))$.

Thus when we pile up the minus signs in front of a, we obtain a or —a alternatively. For the general formula with the appropriate notation, cf.

RULES FOR ADDITION

or, in other words,

N5. $-(a + b) = -a - b$.

Proof. Remember that if x, y are integers, then $x = -y$ and $y = -x$ mean that $x + y = 0$. Thus to prove our assertion, we must show that

$(a -|- b) + (-a - 6) = 0$.

But this comes out immediately, namely,

$(a + b) + (-a - b) = a + b - a - b$

$= a - a + b - b$

$= 0 + 0$

$= 0$.

This proves our formula.

Example. We have

$-(3 + 5) = - 3 - 5 = - 8$,

$-(-4 + 5) = - (- 4) - 5 = 4 - 5 = - 1$,

$-(3 - 7) = -3 - (- 7) = - 3 + 7 = 4$.

You should be very careful when you take the negative of a sum which involves itself in negative numbers, taking into account that

$-(-a) = a$.

The following rule concerning positive integers is so natural that you probably would not even think it worth while to take special notice of it.

We still state it explicitly.

I f a, b are positive integers, then a + b is also a positive integer.

For instance, 17 and 45 are positive integers, and their sum, 62, is also a

positive integer.
We assume this rule concerning positivity. We shall see later that it also applies to positive real numbers. From it we can prove:
If a, b are negative integers, then a + b is negative.
by associativity
by commutativity

Proof. We can write a — —n and b = —m, where m, n are positive.
Therefore
a + b = —n — m = —(ra + m),
which shows that a + b is negative, because n + m is positive.
Example. If we have the relationship between three numbers
a + b = c,
then we can derive other relationships between them. For instance, add —b to both sides of this equation. We get
a + b — b = c — b,
whence a + 0 = c — b, or in other words,
a = c — b.
Similarly, we conclude that
b = c — a.
For instance, if
x + 3 = 5,
then
jc = 5 — 3 = 2 .
If
4 — a = 3,
then adding a to both sides yields
4 = 3 + a,
and subtracting 3 from both sides yields
1 = a.

her words,
N5. —(a + b) = —a — b.
Proof. Remember that if x, y are integers, then x = —y and y = —x mean that x + y = 0. Thus to prove our assertion, we must show that
(a -|- b) + (—a — 6) = 0 .
But this comes out immediately, namely,

$(a + b) + (-a - b) = a + b - a - b$
$= a - a + b - b$
$= 0 + 0$
$= 0.$
This proves our formula.
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In this example we have done something which is frequently useful, namely we have moved to one side all the explicit numbers like 2, 3 and put on the other side those numbers denoted by a letter like a or b. Using commutativity and associativity, we can prove similarly
(5x)(7y) = 35xy
or, with more factors,
(2a)(36)(5x) = SOabx.
We suggest that you carry out the proof of this equality completely, using associativity and commutativity for multiplication.
Finally, we have the rule of distributivity, namely
a{b + c) = ab + ac
and also on the other side,
(6 + c)a = ba + ca.
These rules will not be proved, but will be used constantly. We shall, however, make some comments on them, and prove other rules from them.
First observe that if we just assume distributivity on one side, and commutativity, then we can prove distributivity on the other side. Namely, assuming distributivity on the left, we have
(6 + c)a = a(b + c) = ab + ac = ba + ca,
which is the proof of distributivity on the right.
Observe also that our rule Oa = 0 can be proved from the other rules concerning multiplication and the properties of addition. We carry out the proof as an example. We have
•Oa -I- a = Oa -1- la = (0 l)a = la = a.
Thus
Oa + a = a.

Adding —a to both sides, we obtain
0a + a — a = a — a = 0.

In this example we have done something which is frequently useful, namely we have moved to one side all the explicit numbers like 2, 3 and put on the other side those numbers denoted by a letter like a or b. Using commutativity and associativity, we can prove similarly

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•0a -I- a = 0a -1- la = (0 l)a = la = a.

Thus

0a + a = a.

Adding —a to both sides, we obtain

0a + a — a = a — a = 0.

In each of the above cases, you should indicate specifically each one of the rules we have used to derive the desired equality. Again, we emphasize that you should be especially careful when working with negative numbers and repeated minus signs. This is one of the most frequent sources of error when we work with multiplication and addition.

Example. We have

(-2 a) (36) (4c) = (- 2) - 3 •4a6c

= —24a6c.

Similarly,

(—4*) (530 (—3c) = (—4)5(—3)xyc

= 60xyc.

Note that the product of two minus signs gives a plus sign.

Example. We have

(—1) (—1) = I*

To see this, all we have to do is apply our rule

—(a6) = (—a)6 = a(—6).

We find

(- l) (- l) = - (l (- D) = - (- l) = l.

Example. More generally, for any integers a, 6 we have

N10. (—a) (—6) = a6.

We leave the proof as an exercise. From this we see that a product of two negative numbers is positive, because if a, 6 are positive and —a, —6 are therefore negative, then (—a) (—6) is the positive number a6. For instance, —3 and —5 are negative, but

(—3)(—5) = —(3 (—5)) = —(—(3 •5)) = 15.

Example. A product of a negative number and a positive number is negative. For instance, —4 is negative, 7 is positive, and

(—4) •7 = - (4 - 7) = -2 8 ,

so that (—4) •7 is negative.

When we multiply a number with itself several times, it is convenient to use a notation to abbreviate this operation. Thus we write

aa = a2,

aaa = a3,

aaaa = a4,
and in general if n is a positive integer,
an = aa •••a (the product is taken n times).
We say that an is the n-th power of a. Thus a2 is the second power of a, and a5 is the fifth power of a.
If m, n are positive integers, then
N il. am+n = aman.
This simply states that if we take the product of a with itself m + n times, then this amounts to taking the product of a with itself m times and multiplying this with the product of a with itself n times.
Example
a2a3 = (aa) (aaa) = a2+s = aaaaa = a5.
Example
(Ax)2 = 4x •4x = 4 •4xx = 16x2.
Example
(7x)(2x)(5x) = 1 2 - 5xxx = 70s3.
We have another rule for powers, namely
N12. (am)n = amn.
This means that if we take the product of a with itself m times, and then take the product of amwith itself n times, then we obtain the product of a with itself mn times.
Example. We have
(a 3) 4 = a 12.

Example. We have
(ab)n = anbn
because
(ab)n = abab -a ft (product of ab with itself n times)
= aa •••abb •••b
Example. We have
(2a3)5 = 25(a3)5 = 32a15.
Example. The population of a city is 300 thousand in 1930, and doubles every 20 years. What will be the population after 60 years?

This is a case of applying powers. After 20 years, the population is
2 •300 thousand. After 40 years, the population is 22 •300 thousand. After
60 years, the population is 23 •300 thousand, which is a correct answer.
Of course, we can also say that the population will be 2 million 400 thousand.
The following three formulas are used constantly. They are so important
that they should be thoroughly memorized by reading them out loud and
repeating them like a poemyto get an aural memory of them.

(a + ft)2 = a2 + 2ab + ft2, (a — ft)2 = a2 — 2ab + ft2,

(a + 6) (a — b) = a2 — ft2.

Proofs. The proofs are carried out by applying repeatedly the rules for
multiplication. We have:

(a -f" b)2 = (a + 6)(a + b) = a(a —|—6) —J—6(a -1- b)

= aa -|- ab -I- ba -I- bb

= a2 + ab + ab + b2

= a2 + 2ab + ft2,

which proves the first formula.

(a — b)2 = (a — b) (a — b) = a (a — b) — b(a — b)

= aa — ab — ba + bb

= a2 — ab — ab + b2

= a2 - 2ab + ft2,

which proves the second formula.

(a + b)(a — b) = a(a — 6) + 6(a — b)

which proves the third formula.

Example. We have

(2 + 3x)2 = 22 + 2 •2 •Sx + (3x)2

= 4 + 12x + 9x2.

Example. We have

(3 - 4x)2 = 32 - 2 •3 •4x + (4x)2

= 9 — 24* + 16x2.

Example. We have

(- 2 a + 5b)2 = 4a2 + 2(-2a)(56) + 2562

= 4a2 - 20a6 + 2562.

Example. We have

$(4a - 6) (4a + 6) = (4a)2 - 36$

$= 16a2 - 36.$

We have discussed so far examples of products of two factors. Of course, we can take products of more factors using associativity.

Example. Expand the expression

$(2x + l)(x - 2)(x + 5)$

as a sum of powers of x multiplied by integers.

= aa — ab + ba — bb

= a2 — ab + ab — b2

= a2 - ft2,

We consider the positive integers 1, 2, 3, 4, 5 , . . . , and we shall distinguish between two kinds of integers. We call

1, 3, 5, 7, 9, 11, 13, . . .

the odd integers, and we call

2, 4, 6, 8, 10, 12, 14, . . .

the even integers. Thus the odd integers go up by 2 and the even integers go up by 2. The odd integers start with 1, and the even integers start with 2. Another way of describing an even integer is to say that it is a positive integer which can be written in the form 2n for some positive integer n. For instance, we can write

2 = 2 1 ,

4 = 2-2,

6 = 2-3,

8 = 2-4,

and so on. Similarly, an odd integer is an integer which differs from an even integer by 1, and thus can be written in the form 2m — 1 for some positive integer m. For instance,

1 = 2 1 - 1 ,

3 = 2 •2 - 1,

5 = 2 3 - 1 ,

$7 = 2 * 4 — 1$,

$9 = 2 \cdot 5 - 1$,

and so on. Note that we can also write an odd integer in the form

$2n + 1$

if we allow n to be a natural number, i.e., allowing n = 0. For instance, we have

$1 = 2\ 0 + 1$,

$3 = 2\ 1 + 1$,

$5 = 2 \cdot 2 + 1$,

$7 = 2 \cdot 3 + 1$,

$9 = 2\ 4 + 1$,

and so on.

Theorem 1. Let a, b be positive integers.

I f a is even and b is even, then a + b is even.

I f a is even and b is oddythen a + b is odd.

I f a is odd and b is even, then a + b is odd.

I f a is odd and b is odd, then a + b is even.

Proof. We shall prove the second statement, and leave the others as exercises. Assume that a is even and that b is odd. Then we can write

$a = 2n$ and $b = 2k + 1$

for some positive integer n and some natural number k. Then

a, b — 27i “|“ 2k “I- 1

= 2(n + k) -(-1

$= 2m + 1$ (letting $m = n + k$).

This proves that a + b is odd.

Theorem 2• Let a be a positive integer. I f a is even, then a2 is even. If a is odd, then a2 is odd.

Proof. Assume that a is even. This means that a = 2n for some positive integer n. Then

a2 = 2n •2n = 2(2m2) = 2m,

where m = 2m2 is a positive integer. Thus a2 is even.

Next, assume that a is odd, and write a = 2n + 1 for some natural number n. Then

$a2 = (2 n + l) 2 = (2 m) 2 + 2 (2 m) 1 + l 2$

$= 4m2 + 4m + 1$

$= 2 (2 m 2 + 2 m) + 1$

$= 2 ^ + 1$, where $k = 2 m 2 + 2 m$.

Hence a2 is odd, thus proving our theorem.

Corollary• Let a be a positive integer. I f a2 is even, then a is even. If a2 is odd, then a is odd.

Proof. This is really only a reformulation of the theorem, taking into account ordinary logic. If a2is even, then a cannot be odd because the square of an odd number is odd. If a2 is odd, then a cannot be even because the square of an even number is even.

We can generalize the property used to define an even integer. Let d be a positive integer and let n be an integer. We shall say that d divides n, or that m is divisible by d if we can write

$m = dk$

for some integer k. Thus an even integer is a positive integer which is divisible by 2. According to our definition, the number 9 is divisible by 3 because

$9 = 3 - 3$.

Also, 15 is divisible by 3 because

$15 = 3 \bullet 5$.

Also, —30 is divisible by 5 because

$- 3 0 = 5(—6)$.

Note that every integer is divisible by 1, because we can always write

$n = 1 \bullet 7i$.

Furthermore, every positive integer is divisible by itself

FRACTION

What are Fractions?

Definition 1: A fraction represents a numerical value, which defines the parts of a whole.

Definition 2: A fraction is a number that represents a part of a whole.

Generally, the fraction can be a portion of any quantity out of the whole thing and the whole can be any specific things or value.

The basics of fractions explain the top and bottom numbers of a fraction. The top number represents the number of selected or shaded parts of a whole whereas the bottom number represents the total number of parts.

Suppose a number has to be divided into four parts, then it is represented as x/4. So the fraction here, x/4, defines 1/4th of the number x. Hence, 1/4 is the fraction here. It means one in four equal parts. It can be read as one-fourth or 1/4. This is known as fraction.

Fractions play an important part in our daily lives. There are many examples of fractions you will come across in real life. We have to willingly or unwillingly share that yummy pizza amongst our friends and families. Three people, four slices. If you learn and visualize fractions in an easy way, it will be more fun and exciting. For example, slice an apple into two parts, then each part of the sliced apple will represent a fraction (equal to 1/2).

Parts of Fractions

The fractions include two parts, numerator and denominator.

- **Numerator:** It is the upper part of the fraction, that represents the sections of the fraction
- **Denominator:** It is the lower or bottom part that represents the total parts in which the fraction is divided.

Example: If 3/4 is a fraction, then 3 is the numerator and 4 is the denominator.

Properties of Fractions

Similar to real numbers and whole numbers, a fractional number also holds some of the important properties. They are:

- Commutative and associative properties hold true for fractional addition and multiplication
- The identity element of fractional addition is 0, and fractional multiplication is 1

- The multiplicative inverse of a/b is b/a, where a and b should be non zero elements
- Fractional numbers obey the distributive property of multiplication over addition

Types of Fractions

Based on the properties of numerator and denominator, fractions are sub-divided into different types. They are:

- Proper fractions
- Improper fractions
- Mixed fractions
- Like fractions
- Unlike fractions
- Equivalent fractions

Proper Fractions

The proper fractions are those where the numerator is less than the denominator. For example, 8/9 will be a proper fraction since "numerator < denominator".

Improper Fractions

The improper fraction is a fraction where the numerator happens to be greater than the denominator. For example, 9/8 will be an improper fraction since "numerator > denominator".

Mixed Fractions

A mixed fraction is a combination of the integer part and a proper fraction. These are also called mixed numbers or mixed numerals. For example:

$$3\frac{2}{3} = \frac{[(3 \, x \, 3) + 2]}{3} = \frac{11}{3}$$

Like Fractions

Like fractions are those fractions, as the name suggests, that are alike or same.

For example, take ½ and 2/4; they are alike since if you simplify it mathematically, you will get the same fraction.

Unlike Fractions

Unlike fractions, are those that are dissimilar.

For example, ½ and 1/3 are unlike fractions.

Equivalent Fractions

Two fractions are equivalent to each other if after simplification either of two fractions is equal to the other one.

For example, ⅔ and 4/6 are equivalent fractions.

Since, 4/6 = (2×2)/(2×3) = 2/3

Unit Fractions

A fraction is known as a unit fraction when the numerator is equal to 1.

- One half of whole = ½
- One-third of whole = 1/3
- One-fourth of whole = ¼
- One-fifth of whole = ⅕

Fraction on a Number Line

We have already learned to represent the integers, such as 0, 1, 2, -1, -2, on a number line. In the same way, we can represent fractions on a number line.

= 17/12

Subtracting Fractions

The rule for subtracting two or more fractions is the same as for addition. The denominators should be common to subtract two fractions.

Example: 9/2 – 7/2 = (9-7)/2 = 2/2 = 1

Subtracting with Different Denominators

If the denominators of the two fractions are different, we have to simplify them by finding the LCM of denominators and then making it common for both fractions.

Example: ⅔ – ¾

The two denominators are 3 and 4

Hence, LCM of 3 and 4 = 12

Therefore, multiplying ⅔ by 4/4 and ¾ by 3/3, we get;

8/12 – 9/12

= (8-9)/12

= -1/12

Multiplication of Fractions

As per rule number 2, we have discussed in the previous section, when two fractions are multiplied, then the top part (numerators) and the bottom part (denominators) are multiplied together.

If a/b and c/d are two different fractions, then the multiplication of a/b and c/d will be:

$(a/b) x (c/d) = (axc)/(bxd) = (ac/bd)$

Example: Multiply ⅔ and 3/7.

$(2/3) x (3/7) = (2\times3)/(3\times7) = 2/7$

Division of Fractions

If we have to divide any two fractions, then we will use here rule 3 from the above section, where we need to multiply the first fraction to the reciprocal of the second fraction.

If a/b and c/d are two different fractions, then the division a/b by c/d can be expressed as:

$(a/b)\div(c/d) = (a/b)x(d/c) = (ad/bc)$

Example: Divide ⅔ by 3/7.

$(2/3) \div (3/7) = (2/3) x (7/3) = (2\times7)/(3\times3) = 14/9$

Real-Life Examples of Fractions

Let us visualize some of the fractions examples:

1. Imagine a pie with four slices. Taking two slices of pie for yourself would mean that you have two out of the four. Hence, you represent it as 2/4.

2. Fill half a glass of water. What do you see? 1/2 glass is full. Or 1/2 glass is empty. This 1/2 is fractions where 1 is the numerator that is, the number of parts we have. And 2 is the denominator, the number of parts the whole glass is divided into.

How to Convert Fractions To Decimals?

As we already learned enough about fractions, which are part of a whole. The decimals are the numbers expressed in a decimal form which represents fractions, after division.

For example, Fraction 1/2 can be written in decimal form as 0.5.

The best part of decimals are they can be easily used for any arithmetic operations such as addition, subtraction, etc. Whereas it is difficult sometimes to perform operations on fractions. Let us take an example to understand;

Example: Add 1/6 and 1/4.

solution: 1/6 = 0.17 and 1/4 = 0.25

Hence, on adding 0.17 and 0.25, we get;

0.17 + 0.25 = 0.42

How to Simplify Fractions?

To simplify the fractions easily, first, write the factors of both numerator and denominator. Then find the largest factor that is common for both numerator and denominator. Then divide both the numerator and the denominator by the greatest common factor (GCF) to get the reduced fraction, which is the simplest form of the given fraction. Now, let us consider an example to find the simplest fraction for the given fraction.

For example, take the fraction, 16/48

So, the factors of 16 are 1, 2, 4, 8, 16.

Similarly, the factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, 48.

Thus, the greatest common factor for 16 and 48 is 16.

i.e. GCF (16, 48) = 16.

Now, divide both the numerator and denominator of the given fraction by 16, we get

16/48 = (16/16) / (48/16) = 1/3.

Hence, the simplest form of the fraction 16/48 is 1/3.

Solved Examples on Fractions

Example 1:

Is 12/6 a fraction?

Solution:

Yes, it is. It is called an improper fraction.

Example 2:

Convert 130.1200 into a fraction.

Solution:

Here will use the concept of how to convert decimals into fractions

130.1200 = 130.1200/10000

= 13012/100

Definition of Decimal Fractions

The prerequisite for understanding decimal fractions is the understanding of normal fractions. You must know that a fraction comprises a numerator (top part) and a denominator (bottom part). The correct way of writing a fraction is –X/Y

X is the numerator in this example, and y is the denominator.
Decimal fractions are the fractions in which the denominator (y in the image) must be 10 or a multiple of 10 like 100, 1000, 10000, and so on. The numerator can be any integer (between -infinity and +infinity). These decimal fractions are usually expressed in decimal numbers (numbers with a decimal point).

In algebra, a decimal fraction is a number having 10 or the powers of 10 like 10^1, 10^2, 10^3, and so on in the denominator.

Examples of Decimal Fractions

- 7/10000 is a decimal fraction written in the decimal form as 0.0007.
- 19/10 is a decimal fraction written in the decimal form as 1.9.
- 39/1000 is a decimal fraction written as 0.039.

Reading Decimal Fractions

Let us consider a scenario where 1 is in the numerator. We will consider different denominators to understand how these terms are read with this numerator.

- 1/10 is read as one-tenth.
- 1/100 is read as one-hundredth.
- 1/1000 is read as one-thousandth.

When the value of the numerator is more than one, we add an 's' to the name. So, for instance, 3/10 is read as three-tenths.

History of Decimal Fractions

The Chinese first developed and used decimal fractions at the end of the 4th century BC, which spread to the Middle East before reaching Europe.

Conversion to Decimal Fractions

1. Conversion from fractions to decimal fractions:

- Let us consider an example of a fraction, 3/2.
- The first step would be to consider the number that gives 10 or a multiple of 10 when multiplied by the denominator. In this case, 5 multiplied by 2 gives 10.
- Now multiply the numerator and denominator with the same number to get your decimal fraction. Here, 3 x 5/ 2 x 5 gives 15/10.
- Thus, the decimal fraction of 3/2 is 15/10.

2. Conversion from mixed numbers to decimal fractions:

- Convert the mixed fraction into a normal fraction.
- Follow the steps for converting fractions to decimal fractions.

3. Conversion from decimal numbers to decimal fractions:

- Write the original decimal number in the numerator and denominator form by placing 1 in the denominator: 4.3/1.
- For every space that you move the decimal point, add a zero next to the 1 in the denominator: 43/10 (As we can see one shift of decimal space, one 0 must be added to the denominator).
  4.3/1

  43.0/10

- Once the number in the numerator is non-decimal, you have got your decimal fraction: 4.3 = 43/10.

Real-Life Application of Decimal Fractions

Decimal fractions are used for understanding precise quantities instead of whole numbers. You will also use them for expressing percentages. For instance, 97% can be written as 97/100 for ease of calculation.

Here are some scenarios where you might encounter decimal fractions:

- Coins (They are a fraction of Rupees)
- Weighing products
- Measuring ingredients while cooking

How to find HCF and LCM?

Here are the methods we can use to find the HCF and LCM of given numbers.

1. Prime factorization method
2. Division method

Let us learn both methods, one by one.

HCF by Prime Factorization Method

Take an example of finding the highest common factor of 144, 104 and 160.
Now let us write the prime factors of 144, 104 and 160.
144 = 2 × 2 × 2 × 2 × 3 × 3
104 = 2 × 2 × 2 × 13
160 = 2 × 2 × 2 × 2 × 2 × 5
The common factors of 144, 104 and 160 are 2 × 2 × 2 = 8
Therefore, HCF (144, 104, 160) = 8

HCF by Division Method

Steps to find the HCF of any given numbers;

1) Larger number/ Smaller Number
2) The divisor of the above step / Remainder
3) The divisor of step 2 / remainder. Keep doing this step till R = 0(Zero).
4) The last step's divisor will be HCF.

The above steps can also be used to find the HCF of more than 3 numbers.
Example: Find the HCF of 144 and 160 by division method.
Since 160>144, so the dividend will be 160 and the divisor will be 144.
By using the division method, we get:

```
144) 160 (1
     144
-----------------
     16) 144 (9
         144
-----------------
         XXX
-----------------
```

Hence, we can see here that 16 is the highest number which divides 160 and 144.
Therefore, HCF (144, 160) = 16

LCM by Prime Factorization Method

To calculate the LCM of two numbers 60 and 45. Out of other ways, one way to find the LCM of given numbers is as below:

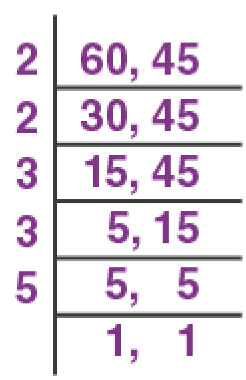
- List the **prime factors** of each number first.
  60 = 2 × 2 x 3 × 5
  45 = 3 × 3 × 5

- Then multiply each factor the **most number of times** it occurs in any number.

If the same multiple occurs more than once in both the given numbers, then multiply the factor by the most number of times it occurs.
The occurrence of Numbers in the above example:
2: two times
3: two times
5: one time
LCM = 2 × 2 x 3 × 3 × 5 = 180

LCM by Division Method

Let us see with the same example, which we used to find the LCM using prime factorization.
Solve LCM of (60,45) by division method.



Therefore, LCM of 60 and 45 = 2 × 2 x 3 × 3 × 5 = 180
At BYJU'S you can also learn, Prime Factorization Of Hcf And Lcm.

HCF and LCM Examples

Example 1:

Find the Highest Common Factor of 25, 35 and 45.
Solution:

Given, three numbers as 25, 35 and 45.

We know, 25 = 5 × 5

35 = 5 × 7

45 = 5 × 9

From the above expression, we can say 5 is the only common factor for all three numbers.

Therefore, 5 is the HCF of 25, 35 and 45.

Example 2:

Find the Least Common Multiple of 36 and 44.

Number	Square root (√)	Cube root (∛)
1	1.000	1.000
2	1.414	1.260
3	1.732	1.442
4	2.000	1.587
5	2.236	1.710
6	2.449	1.817
7	2.646	1.913
8	2.828	2.000
9	3.000	2.080
10	3.162	2.154
11	3.317	2.224

12	3.464	2.289
13	3.606	2.351
14	3.742	2.410
15	3.873	2.466

Solution:

Given, two numbers 36 and 44. Let us find out the LCM, by division method.

Square root and Cube roots are the most important topics in Maths, especially for class 8 students. To find the *square root* of any number, we need to find a number which when multiplied twice by itself gives the original number. Similarly, to find the cube root of any number we need to find a number which when multiplied three times by itself gives the original number.

Symbol: The square root is denoted by the symbol '√', whereas the cube root is denoted by '∛'.

Examples:

- $\sqrt{4} = \sqrt{(2 \times 2)} = 2$
- $\sqrt[3]{27} = \sqrt[3]{(3 \times 3 \times 3)} = 3$

Square Root and Cube Root Table

Memorizing the squares and the square roots of the first few numbers are almost elementary and it can help you to solve problems much faster rather than having to work on it. Following is the **square roots list** and **Cube root list** of the first 15 natural numbers.

How to find Square Root and Cube Root

To find the square root of the number, we have to determine which number was squared to get the original number. For example, if we have to find the root of 16, then as we know, when we multiply 4 by 4, the result is 16. Hence, $\sqrt{16} = 4$. Similarly, if we have to find the cube root of a number, say 64, then it is easy to determine that the cube of 4 gives 64. So the cube root of 64 is 4. But if the numbers are very large, then to find the roots, we have to use the prime factorisation method. Let us see some examples.

Solved Examples

Example 1: Find Square root of 256.

Solution: Given: The number is 256.

Prime factorisation of $256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = (2 \times 2 \times 2 \times 2)^2 = 16^2$

Taking roots on both the sides, we get;

$\sqrt{256} = 16$.

Hence, 16 is the a

RATIO

Ratio, in math, is a term that is used to compare two or more numbers. It is used to indicate how big or small a quantity is when compared to another. In a ratio, two quantities are compared using division. Here the dividend is called the 'antecedent' and the divisor is called the 'consequent'. For example, in a group of 30 people, 17 of them prefer to walk in the morning and 13 of them prefer to cycle. To represent this information as a ratio, we write it as 17: 13. Here, the symbol ': ' is read as "is to". So, the ratio of people who prefer walking to the people who prefer cycling is read as '17 is to 13'.

WHAT IS RATIO?

The ratio is defined as the comparison of two quantities of the same units that indicates how much of one quantity is present in the other quantity. Ratios can be classified into two types. One is part to part ratio and the other is part to whole ratio. The part-to-part ratio denotes how two distinct entities or groups are related. For example, the ratio of boys to girls in a class is 12: 15, whereas, the part-to-whole ratio denotes the relationship between a specific group to a whole. For example, out of every 10 people, 5 of them like to read books. Therefore, the part to the whole ratio is 5: 10, which means every 5 people from 10 people like to read books.

RATIO FORMULA

We use the ratio formula while comparing the relationship between two numbers or quantities. The general form of representing a ratio of between two quantities say 'a' and 'b' is **a: b,** which is read as **'a is to b'.**

The fraction form that represents this ratio is a/b. To further simplify a ratio, we follow the same procedure that we use for simplifying a fraction. a:b = a/b. Let us understand this with an example.

Example: In a class of 50 students, 23 are girls and the remaining are boys. Find the ratio of the number of boys to the number of girls.

Total number of students = 50; Number of girls = 23.

Total number of boys = Total number of students - Total number of girls
= 50 - 23
= 27

Therefore, the desired ratio is, (Number of boys: Number of girls), which is **27:23.**

Calculation of Ratios

In order to calculate the ratio of two quantities, we can use the following steps. Let us understand this with an example. For example, if 15 cups of flour and 20 cups of sugar are needed to make fluffy pancakes, let us calculate the ratio of flour and sugar used in the recipe.

- **Step 1:** Find the quantities of both the scenarios for which we are determining the ratio. In this case, it is 15 and 20.
- **Step 2:** Write it in the fraction form a/b. So, we write it as 15/20.
- **Step 3:** Simplify the fraction further, if possible. The simplified fraction will give the final ratio. Here, 15/20 can be simplified to 3/4.
- **Step 4:** Therefore, the ratio of flour to sugar can be expressed as 3: 4.

Use Cuemath's free online ratios calculator to verify your answers while calculating ratios.

HOW TO SIMPLIFY RATIOS?

A ratio expresses how much of one quantity is required as compared to another quantity. The two terms in the ratio can be simplified and expressed in their lowest form. Ratios when expressed in their lowest terms are easy to understand and can be simplified in the same way as we simplify fractions. To simplify a ratio, we use the following steps. Let us understand this with an example. For example, let us simplify the ratio 18:10.

- **Step 1:** Write the given ratio a:b in the form of a fraction a/b. On writing the ratio in the fraction form, we get 18/10.
- **Step 2:** Find the greatest common factor of 'a' and 'b'. In this case, the GCF of 10 and 18 is 2.
- **Step 3:** Divide the numerator and denominator of the fraction with the GCF to obtain the simplified fraction. Here, by dividing the numerator and denominator by 2, we get, (18÷2)/(10÷2) = 9/5.
- **Step 4:** Represent this fraction in the ratio form to get the result. Therefore, the simplified ratio is 9:5.

Use Cuemath's free online simplifying ratios calculator to verify your answers.

Tips and Tricks on Ratio:

- In case both the numbers 'a' and 'b' are equal in the ratio a: b, then a: b = 1.
- If a > b in the ratio a : b, then a : b > 1.
- If a < b in the ratio a : b, then a : b < 1.
- It is to be ensured that the units of the two quantities are similar before comparing them.

Equivalent Ratios

Equivalent ratios are similar to equivalent fractions. If the antecedent (the first term) and the consequent (the second term) of a given ratio are multiplied or divided by the same number other than zero, it gives an equivalent ratio. For example, when the antecedent and the consequent of the ratio 1:3 are multiplied by 3, we get, (1 × 3) : (3 × 3) or 3: 9. Here, 1:3 and 3:9 are equivalent ratios. Similarly, when both the terms of the ratio 20:10 are divided by 10, it gives 2:1. Here, 20:10 and 2:1 are equivalent ratios. An infinite number of equivalent ratios of any given ratio can be found by multiplying the antecedent and the consequent by a positive integer.

RATIO TABLE

A **ratio table** is a list containing the equivalent ratios of any given ratio in a structured manner. The following ratio table gives the relation between the ratio 1:4 and four of its equivalent ratios. The equivalent ratios are related to each other by the multiplication of a number. Equivalent ratios are obtained by multiplying or dividing the two terms of a ratio by the same number. In the example shown in the figure, let us take the ratio 1:4 and find four equivalent ratios, by multiplying both the terms of the ratio by 2, 3, 6, and 9. As a result, we get 2:8, 3:12, 6:24, and 9:36.

The term 'per cent' means 'out of a hundred'. In mathematics, percentages are used like fractions and decimals, as ways to describe parts of a whole. When you are using percentages, the whole is considered to be made up of a hundred equal parts. The symbol % is used to show that a number is a percentage, and less commonly the abbreviation 'pct' may be used.

You will see percentages almost everywhere: in shops, on the internet, in advertisements and in the media. Being able to understand what percentages mean is a key skill that will potentially save you time and money and will also make you more employable.

The Meaning of Percentages

Percentage is a term from Latin, meaning ‘out of one hundred’.

You can therefore consider each ‘whole’ as broken up into 100 equal parts, each one of which is a single percent.

The box below shows this for a simple grid, but it works the same way for anything: children in a class, prices, pebbles on the beach, and so on.

It is easy to work out the percentage when there are 100 individual ‘things’ making up the whole, as in the grid above. But what if there are more or less?

The answer is that you **convert** the individual elements that make up the whole into a percentage. For example, if there had been 200 cells in the grid, each percentage (1%) would be two cells, and every cell would be half a percent.

We use percentages to make calculations easier. It is much simpler to work with parts of 100 than thirds, twelfths and so on, especially because quite a lot of fractions do not have an exact (non-recurring) decimal equivalent. Importantly, this also makes it much easier to make comparisons between percentages (which all effectively have the common denominator of 100) than it is between fractions with different denominators. This is partly why so many countries use a metric system of measurement and decimal currency.

Finding the Percentage

The general rule for finding a given percentage of a given whole is:

Work out the value of 1%, then multiply it by the percentage you need to find.

This is easiest to understand with an example. Let’s suppose that you want to buy a new laptop computer. You have checked local suppliers and one company has offered to give you 20% off the list price of £500. How much will the laptop cost from that supplier?

In this example, the whole is £500, or the cost of the laptop before the discount is applied. The percentage that you need to find is 20%, or the discount offered by the supplier. You are then going to take that off the full price to find out what the laptop will cost you.

1. **Start by working out the value of 1%**

   One percent of £500 is £500 ÷ 100 = £5.

2. **Multiply it by the percentage you are looking for**

   Once you have worked out the value of 1%, you simply multiply it by the percentage you are looking for, in this case 20%.

   £5 × 20 = £100.

   You now know that the discount is worth £100.

3. **Complete the calculation by adding or subtracting as necessary.**

   The price of the laptop, including the discount, is £500−20%, or £500−£100 = **£400**.

WHAT IS TRIGONOMETRY?

Trigonometry is one of the most important branches in mathematics that finds huge application in diverse fields. The branch called "Trigonometry" basically deals with the study of the relationship between the sides and angles of the right-angle triangle. Hence, it helps to find the missing or unknown angles or sides of a right triangle using the trigonometric formulas, functions or trigonometric identities. In trigonometry, the angles can be either measured in degrees or radians. Some of the most commonly used trigonometric angles for calculations are 0°, 30°, 45°, 60° and 90°.

Trigonometry is further classified into two sub-branches. The two different types of trigonometry are:

Plane Trigonometry

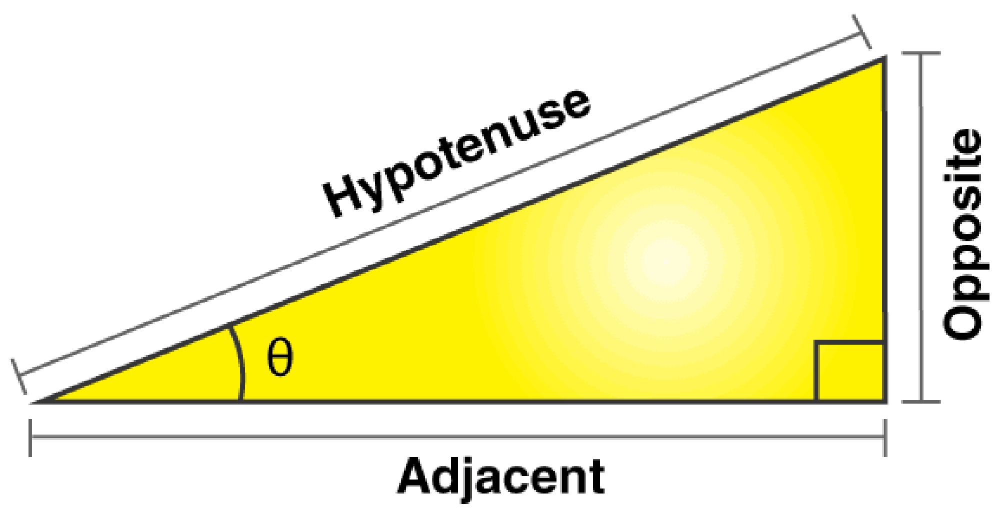
Spherical Trigonometry

In this article, let us discuss the six important trigonometric functions, ratios, trigonometry table, formulas and identities which helps to find the missing angles or sides of a right triangle.

Trigonometry Ratios-Sine, Cosine, Tangent

The trigonometric ratios of a triangle are also called the trigonometric functions. Sine, cosine, and tangent are 3 important trigonometric functions and are abbreviated as sin, cos and tan. Let us see how are these ratios or functions, evaluated in case of a right-angled triangle.

Consider a right-angled triangle, where the longest side is called the hypotenuse, and the sides opposite to the hypotenuse are referred to as the adjacent and opposite sides.



Six Important Trigonometric Functions

The six important trigonometric functions (trigonometric ratios) are calculated using the below formulas and considering the above figure. It is necessary to get knowledge about the sides of the right triangle because it defines the set of important trigonometric functions.

Functions	**Abbreviation**	**Relationship to sides of a right triangle**
Sine Function	sin	Opposite side/ Hypotenuse
Tangent Function	tan	Opposite side / Adjacent side

Cosine Function	cos	Adjacent side / Hypotenuse
Cosecant Function	cosec	Hypotenuse / Opposite side
Secant Function	sec	Hypotenuse / Adjacent side
Cotangent Function	cot	Adjacent side / Opposite side

Even and Odd Trigonometric Functions

The trigonometric function can be described as being even or odd.

Odd trigonometric functions: A trigonometric function is said to be an odd function if f(-x) = -f(x) and symmetric with respect to the origin.

Even trigonometric functions: A trigonometric function is said to be an even function, if f(-x) = f(x) and symmetric to the y-axis.

We know that

- Sin (-x) = – Sin x
- Cos (-x) = Cos x
- Tan (-x) = -Tan x
- Csc (-x) = – Csc x
- Sec (-x) = Sec x
- Cot (-x) = -Cot x

Therefore, cosine and secant are the even trigonometric functions, whereas sine, tangent, cosecant and cotangent are the odd trigonometric functions. If we know the even and odd trigonometric functions, it helps us to simplify the trigonometric expression when the variable inside the trigonometric function is negative.

Trigonometry Angles

The trigonometry angles which are commonly used in trigonometry problems are 0°, 30°, 45°, 60° and 90°**.** The trigonometric ratios such as sine, cosine and tangent of these angles are easy to memorize. We will also show the table where all the ratios and their respective angle's values are mentioned. To find these angles we have to draw a right-angled triangle, in which one of the acute angles will be the corresponding trigonometry angle. These angles will be defined with respect to the ratio associated with it.

For example, in a right-angled triangle,

Sin θ = Perpendicular/Hypotenuse

or $\theta = \sin^{-1}$ (P/H)

Similarly,

$\theta = \cos^{-1}$ (Base/Hypotenuse)

$\theta = \tan^{-1}$ (Perpendicular/Base)

Trigonometry Table

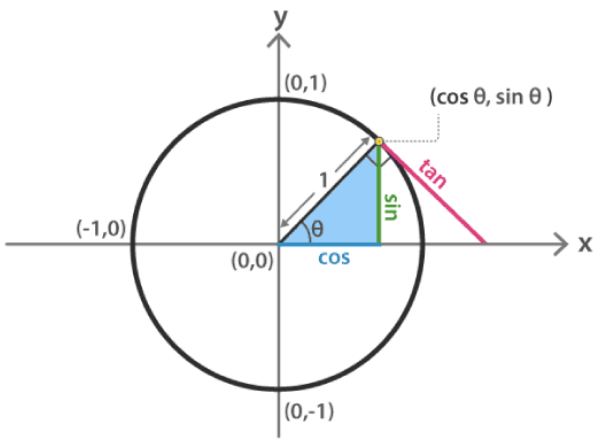
Check the table for common angles which are used to solve many trigonometric problems involving trigonometric ratios.

Angles	**0°**	**30°**	**45°**	**60°**	**90°**
Sin θ	0	½	1/√2	√3/2	1
Cos θ	1	√3/2	1/√2	½	0
Tan θ	0	1/√3	1	√3	∞
Cosec θ	∞	2	√2	2/√3	1
Sec θ	1	2/√3	√2	2	∞
Cot θ	∞	√3	1	1/√3	0

In the same way, we can find the trigonometric ratio values for angles beyond 90 degrees, such as 180°, 270° and 360°.

UNIT CIRCLE

The concept of unit circle helps us to measure the angles of cos, sin and tan directly since the centre of the circle is located at the origin and radius is 1. Consider theta be an angle then,



Suppose the length of the perpendicular is y and of base is x. The length of the hypotenuse is equal to the radius of the unit circle, which is 1. Therefore, we can write the trigonometry ratios as;

Sin θ	y/1 = y
Cos θ	x/1 = x
Tan θ	y/x

List of Trigonometry Formulas

The Trigonometric formulas or Identities are the equations which are true in the case of Right-Angled Triangles. Some of the special trigonometric identities are given below –

1. **Pythagorean Identities**

- $\sin^2\theta + \cos^2\theta = 1$
- $\tan^2\theta + 1 = \sec^2\theta$
- $\cot^2\theta + 1 = \mathrm{cosec}^2\theta$
- $\sin 2\theta = 2 \sin \theta \cos \theta$
- $\cos 2\theta = \cos^2\theta - \sin^2\theta$
- $\tan 2\theta = 2 \tan \theta / (1 - \tan^2\theta)$

- $\cot 2\theta = (\cot^2\theta - 1) / 2 \cot \theta$

2. **Sum and Difference identities-**

For angles u and v, we have the following relationships:

- $\sin(u + v) = \sin(u)\cos(v) + \cos(u)\sin(v)$
- $\cos(u + v) = \cos(u)\cos(v) - \sin(u)\sin(v)$
- $\sin(u - v) = \sin(u)\cos(v) - \cos(u)\sin(v)$
- $\cos(u - v) = \cos(u)\cos(v) + \sin(u)\sin(v)$

3. **If A, B and C are angles and a, b and c are the sides of a triangle, then,**

Sine Laws

- $a/\sin A = b/\sin B = c/\sin C$

Cosine Laws

- $c^2 = a^2 + b^2 - 2ab \cos C$
- $a^2 = b^2 + c^2 - 2bc \cos A$
- $b^2 = a^2 + c^2 - 2ac \cos B$

Trigonometry Identities

The three important trigonometric identities are:

- $\sin^2\theta + \cos^2\theta = 1$
- $\tan^2\theta + 1 = \sec^2\theta$
- $\cot^2\theta + 1 = \text{cosec}^2\theta$

Euler's Formula for trigonometry

As per the euler's formula,

$e^{ix} = \cos x + i \sin x$

Trigonometry Basics

The three basic functions in trigonometry are sine, cosine and tangent. Based on these three functions the other three functions that are cotangent, secant and cosecant are derived.

All the trigonometrically concepts are based on these functions. Hence, to understand trigonometry further we need to learn these functions and their respective formulas at first.

If θ is the angle in a right-angled triangle, then

Sin θ = Perpendicular/Hypotenuse

Cos θ = Base/Hypotenuse

Tan θ = Perpendicular/Base

Perpendicular is the side opposite to the angle θ.

The base is the adjacent side to the angle θ.

The hypotenuse is the side opposite to the right angle

The other three functions i.e. cot, sec and cosec depend on tan, cos and sin respectively, such as:

Cot θ = 1/tan θ

Sec θ = 1/cos θ

Cosec θ = 1/sin θ

Hence,

Cot θ = Base/Perpendicular

Sec θ = Hypotenuse/Base

Cosec θ = Hypotenuse/Perpendicular

Trigonometry Examples

There are many real-life examples where trigonometry is used broadly.

If we have been given with height of the building and the angle formed when an object is seen from the top of the building, then the distance between object and bottom of the building can be determined by using the tangent function, such as tan of angle is equal to the ratio of the height of the building and the distance. Let us say the angle is ∝, then

Tan ∝ = Height/Distance between object & building

Distance = Height/Tan ∝

Let us assume that height is 20m and the angle formed is 45 degrees, then

Distance = 20/Tan 45°

Since, tan 45° = 1

So, Distance = 20 m

Applications of Trigonometry

- Its applications are in various fields like oceanography, seismology, meteorology, physical sciences, astronomy, acoustics, navigation, electronics, etc.
- It is also helpful to measure the height of the mountain, find the distance of long rivers, etc.

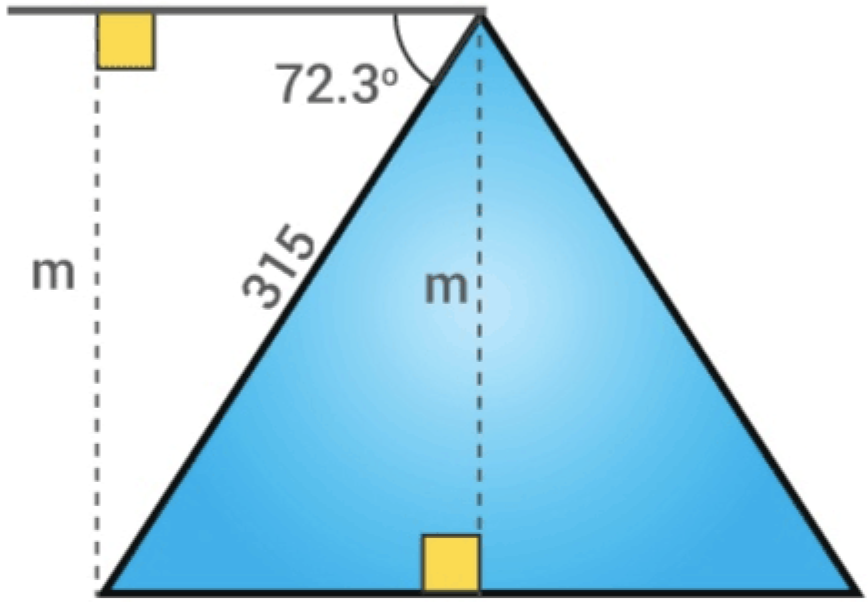
Video Lesson on Applications of Trigonometry

Trigonometry Problems and Solutions

Example 1: Two friends, Rakesh and Vishal started climbing a pyramid-shaped hill. Rakesh climbs 315 m and finds that the angle of depression is 72.3 degrees from his starting point. How high is he from the ground?

Solution: Let m is the height above the ground.

To find: Value of m



To solve m, use the sine ratio.

Sin 72.3° = m/315

0.953 = m/315

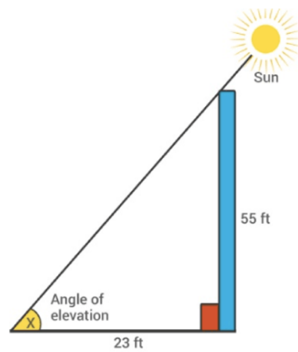
m= 315 x 0.953

m=300.195 mtr

The man is 300.195 mtr above the ground.

Example 2: A man is observing a pole of height 55 foot. According to his measurement, pole cast a 23 feet long shadow. Can you help him to know the angle of elevation of the sun from the tip of shadow?

Solution:



Let x be the angle of elevation of the sun, then

tan x = 55/23 = 2.391

x = $\tan^{-1}$(2.391)

or x = 67.30 degrees

Trigonometry Questions

Practise these questions given here to get a deep knowledge of Trigonometry. Use the formulas and table given in this article wherever necessary.

Q.1: In △ABC, right-angled at B, AB=22 cm and BC=17 cm. Find:

(a) sin A Cos B

(b) tan A tan B

Q.2: If 12cot θ= 15, then find sec θ.

Q.3: In Δ PQR, right-angled at Q, PR + QR = 30 cm and PQ = 10 cm. Determine the values of sin *P,* cos *P* and tan *P.*

Q.4: If sec 4θ = cosec (θ- 30^0), where 4θ is an acute angle, find the value of A.

Frequently Asked Questions on Trigonometry

What do you Mean by Trigonometry?

Trigonometry is one of the branches of mathematics which deals with the relationship between the sides of a triangle (right triangle) with its angles. There are 6 trigonometric functions for which the relation between sides and angles are defined. Learn more about trigonometry now by visiting BYJU'S.

What are the six basic Trigonometric Functions?

There are 6 trigonometric functions which are:

- Sine function
- Cosine function
- Tan function
- Sec function
- Cot function
- Cosec function

What is the formula for six trigonometry functions?

The formula for six trigonometry functions are:
Sine A = Opposite side/Hypotenuse
Cos A = Adjacent side / Hypotenuse
Tan A = Opposite side / Adjacent side
Cot A = Adjacent side / Opposite side
Sec A = Hypotenuse / Adjacent side
Cosec A = Hypotenuse / Opposite side

What is the primary function of trigonometry?

The three primary functions of trigonometry are Sine function, Cosine function and Tangent Function.

Who is the founder of trigonometry?

A greek astronomer, geographer and mathematician, Hipparchus discovered the concept of trigonometry.

What are the Applications of Trigonometry in Real Life?

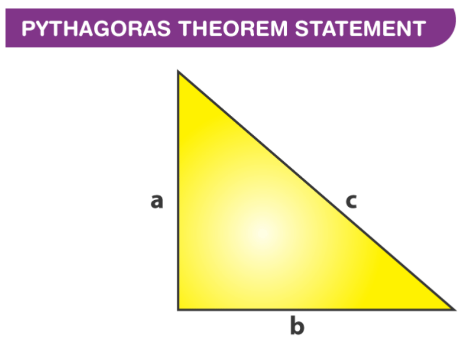
One of the most important real-life applications of trigonometry is in the calculation of height and distance. Some of the sectors where the concepts of trigonometry are extensively used are aviation department, navigation, criminology, marine biology, etc. Learn more about the applications of trigonometry here.

Learn about Trigonometry in a simple manner with detailed information, along with step by step solutions to all questions, only at BYJU'S. Download the app to get personalised videos.

Pythagoras Theorem Statement

Pythagoras theorem states that "**In a right-angled triangle, the square of the hypotenuse side is equal to the sum of squares of the other two sides**". The sides of this triangle have been named Perpendicular, Base and Hypotenuse. Here, the **hypotenuse** is the longest side, as it is opposite to the angle 90°. The sides of a right triangle (say a, b and c) which have positive integer values, when squared, are put into an

equation, also called a Pythagorean triple.



History

The theorem is named after a Greek Mathematician called Pythagoras.

Pythagoras Theorem Formula

Consider the triangle given above:

Where "a" is the perpendicular,

"b" is the base,

"c" is the hypotenuse.

According to the definition, the Pythagoras Theorem formula is given as:

$Hypotenuse^2 = Perpendicular^2 + Base^2$

$c^2 = a^2 + b^2$

The side opposite to the right angle (90°) is the longest side (known as Hypotenuse) because the side opposite to the greatest angle is the longest.

Consider three squares of sides a, b, c mounted on the three sides of a triangle having the same sides as shown.

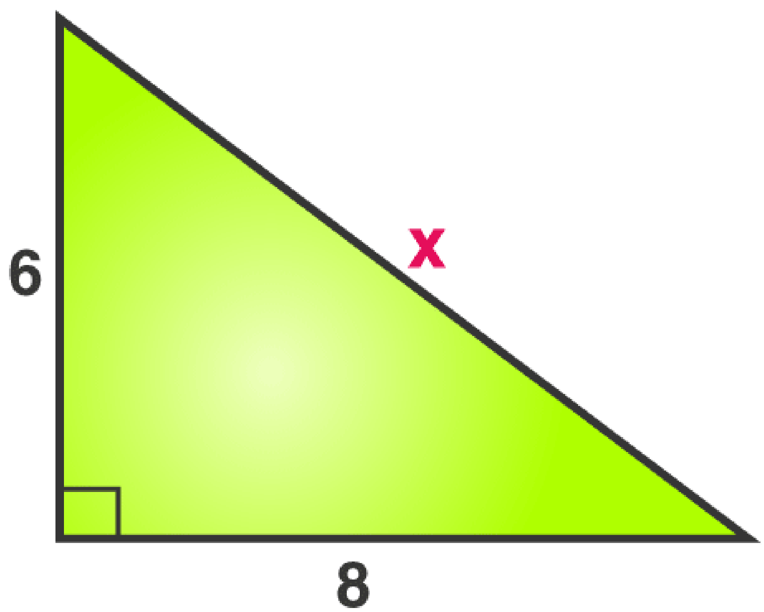
By Pythagoras Theorem –

Area of square "a" + Area of square "b" = Area of square "c"

Example

The examples of theorem and based on the statement given for right triangles is given below:

Consider a right triangle, given below:



Find the value of x.

X is the side opposite to the right angle, hence it is a hypotenuse.

Now, by the theorem we know;

$\text{Hypotenuse}^2 = \text{Base}^2 + \text{Perpendicular}^2$

$x^2 = 8^2 + 6^2$

$x^2 = 64+36 = 100$

$x = \sqrt{100} = 10$

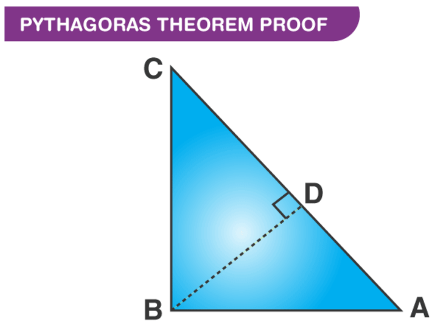
Therefore, the value of x is 10.

Pythagoras Theorem Proof

Given: A right-angled triangle ABC, right-angled at B.

To Prove- $AC^2 = AB^2 + BC^2$

Construction: Draw a perpendicular BD meeting AC at D.



Proof:

We know, △ADB ~ △ABC

Therefore,

$$\frac{AD}{AB}=\frac{AB}{AC}$$

(corresponding sides of similar triangles)

Or, $AB^2 = AD \times AC$ ……………………………..……..(1)

Also, △BDC ~△ABC

Therefore,

$$\frac{CD}{BC}=\frac{BC}{AC}$$

(corresponding sides of similar triangles)

Or, $BC^2 = CD \times AC$ ……………………………………..(2)

Adding the equations (1) and (2) we get,

$AB^2 + BC^2 = AD \times AC + CD \times AC$

$AB^2 + BC^2 = AC (AD + CD)$

Since, AD + CD = AC

Therefore, $AC^2 = AB^2 + BC^2$

Hence, the Pythagorean theorem is proved.

Note: Pythagorean theorem is only applicable to Right-Angled triangle.

Coordinate Geometry

Coordinate Geometry is considered to be one of the most interesting concepts of mathematics. Coordinate Geometry (or the analytic geometry) describes the link between geometry and algebra through graphs involving curves and lines. It provides geometric aspects in Algebra and enables them to solve geometric problems. It is a part of geometry where the position of points on the plane is described using an ordered pair of numbers. Here, the concepts of coordinate geometry (also known as Cartesian geometry) are explained along with its formulas and their derivations.

Introduction to Coordinate Geometry

Coordinate geometry (or analytic geometry) is defined as the study of geometry using the coordinate points. Using coordinate geometry, it is possible to find the distance between two points, dividing lines in m:n ratio, finding the mid-point of a line, calculating the area of a triangle in the Cartesian plane, etc. There are certain terms in Cartesian geometry that should be properly understood. These terms include:

Coordinate Geometry Definition	It is one of the branches of geometry where the position of a point is defined using coordinates.
What are the Coordinates?	Coordinates are a set of values which helps to show the exact position of a point in the coordinate plane.
Coordinate Plane Meaning	A coordinate plane is a 2D plane which is formed by the intersection of two perpendicular lines known as the x-axis and y-axis.
Distance	It is used to find the distance

Formula	between two points situated in $A(x_1,y_1)$ and $B(x_2,y_2)$
Section Formula	It is used to divide any line into two parts, in m:n ratio
Mid-Point Theorem	This formula is used to find the coordinates at which a line is divided into two equal halves.

What is a Co-ordinate and a Co-ordinate Plane?

You must be familiar with plotting graphs on a plane, from the tables of numbers for both linear and non-linear equations. The number line which is also known as a Cartesian plane is divided into four quadrants by two axes perpendicular to each other, labelled as the x-axis (horizontal line) and the y-axis(vertical line).

The four quadrants along with their respective values are represented in the graph below-

- Quadrant 1 : (+x, +y)
- Quadrant 2 : (-x, +y)
- Quadrant 3 : (-x, -y)
- Quadrant 4 : (+x, -y)

The point at which the axes intersect is known as the **origin**. The location of any point on a plane is expressed by a pair of values (x, y) and these pairs are known as the **coordinates**.

The figure below shows the Cartesian plane with coordinates (4,2). If the coordinates are identified, the distance between the two points and the interval's midpoint that is connecting the points can be computed.

Equation of a Line in Cartesian Plane

Equation of a line can be represented in many ways, few of which is given below-

(i) General Form

The general form of a line is given as $Ax + By + C = 0$.

(ii) Slope intercept Form

Let x, y be the coordinate of a point through which a line passes, m be the slope of a line, and c be the y-intercept, then the equation of a line is given by:

$$y=mx + c$$

(iii) Intercept Form of a Line

Consider a and b be the x-intercept and y-intercept respectively, of a line, then the equation of a line is represented as-

$$y = mx + c$$

Slope of a Line:

Consider the general form of a line $Ax + By + C = 0$, the slope can be found by converting this form to the slope-intercept form.

$$Ax + By + C = 0$$

$$\Rightarrow By = -Ax - C$$

or,

Comparing the above equation with $y = mx + c$,

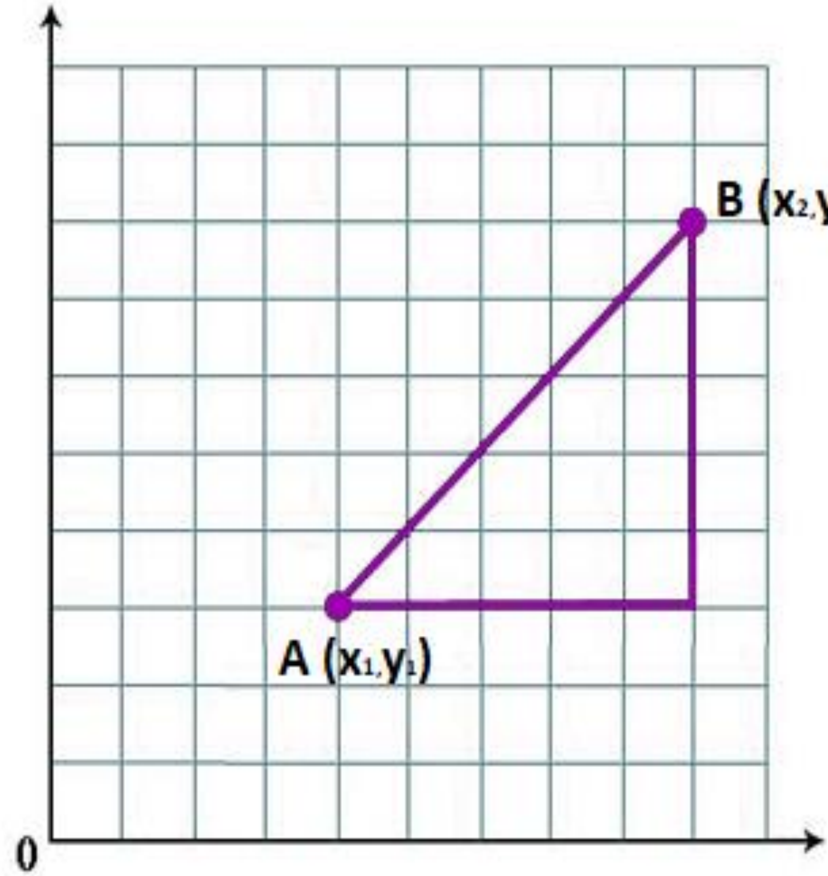
Thus, we can directly find the slope of a line from the general equation of a line.

Coordinate Geometry Formulas and Theorems

Distance Formula: To Calculate Distance Between Two Points

Let the two points be A and B, having coordinates to be (x_1, y_1) and (x_2, y_2), respectively.

Thus, the distance between two points is given as-



Coordinate Geometry Fig. 2: Distance Formula

Midpoint Theorem: To Find Mid-point of a Line Connecting Two Points

Consider the same points A and B, which have coordinates (x_1, y_1) and (x_2, y_2), respectively. Let M(x,y) be the midpoint of lying on the line connecting these two points A and B. The coordinates of point M is given as-

Angle Formula: To Find The Angle Between Two Lines

Consider two lines A and B, having their slopes m_1 and m_2, respectively.

Let **"θ"** be the angle between these two lines, then the angle between them can be represented as-

Area of a Triangle in Cartesian Plane

The area of a triangle In coordinate geometrywhose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is

If the area of a triangle whose vertices are (x_1, y_1),(x_2, y_2) and (x_3, y_3) is zero, then the three points are collinear.

- **Important:** Click here to Download Co-ordinate Geometry pdf

Examples Based On Coordinate Geometry Concepts

Coordinate Geometry Questions For Practice

1. Calculate the ratio in which the line 2x + y – 4 = 0 divides the line segment joining the points A(2, – 2) and B(3, 7).
2. Find the area of the triangle having vertices at A, B, and C which are at points (2, 3), (−1, 0), and (2, – 4), respectively. Also, mention the type of triangle.
3. A point A is equidistant from B(3, 8) and C(-10, x). Find the value for x and the distance BC.

SURFACE AREA AND VOLUME

Surface area and volume are calculated for any three-dimensional geometrical shape. The surface area of any given object is the area or region occupied by the surface of the object. Whereas volume is the amount of space available in an object.

In geometry, there are different shapes and sizes such as sphere, cube, cuboid, cone, cylinder, etc. Each shape has its surface area as well as volume. But in the case of two-dimensional figures like square, circle, rectangle, triangle, etc., we can measure only the area covered by these figures and there is no volume available. Now, let us see the formulas of surface areas and volumes for different 3d-shapes.

What is Surface Area?

The space occupied by a two-dimensional flat surface is called the area. It is measured in square units. The area occupied by a three-dimensional object by its outer surface is called the surface area. It is also measured in square units.

Generally, Area can be of two types:

(i) Total Surface Area

(ii) Curved Surface Area/Lateral Surface Area

Total Surface Area

Total surface area refers to the area including the base(s) and the curved part. It is the total area covered by the surface of the object. If the shape has a curved surface and base, then the total area will be the sum of the two areas.

Curved Surface Area/Lateral Surface Area

Curved surface area refers to the area of only the curved part of the shape excluding its base(s). It is also referred to as lateral surface area for shapes such as a cylinder.

What is Volume?

The amount of space, measured in cubic units, that an object or substance occupies is called volume. Two-dimensional doesn't have volume but has area only. For example, the **Volume of the Circle** cannot be found, though the Volume of the sphere can be. It is so because a sphere is a three-dimensional shape.

Learn more: Mathematics Grade 10

Surface Area and Volume Formulas

Below given is the *table* for calculating **Surface area and Volume** for the basic geometrical figures:

Related Articles

- Surface Area and Volume Class 9
- Surface Areas and Volumes Class 10 Notes

Also have a look on:

Frequently Asked Questions on Surface Area and Volume

What are the formulas for surface area and volume of cuboid?

Surface area of cuboid = 2(lb+bh+hl)
Volume = l × b × h
where l = length, b=breadth and h = height.

What is the total surface area of the cylinder?

The total surface area of the cylinder = 2 π r(r+h), where r is the radius of the circular base and h is the height of the cylinder.

How to calculate the volume of a cone-shaped object?

If r is the radius of the circular base of the cone-shaped object and h is the height, then the formula to find the volume of the cone is given by: $V = 1/3\pi r^2 h$

What is the total surface area of the hemisphere?

The total surface area of the hemisphere is equal to the sum of half of the surface area of sphere and the area of its circular base.
Total surface area of hemisphere = $2 \pi r^2 + \pi r^2 = 3 \pi r^2$

System of Units

- Post authorBy Hemant More
- Post dateMarch 3, 2020
- 13 Commentson System of Units

There are as many units as there are independent quantities. We consider length, mass, and time three quantities which are independent of each other. Hence they have three separate units for their measurements. Hence it is required to define systems of units.

A system of units is a collection of units in which certain units are chosen as fundamental and all others are derived from them. This system is also called an absolute system of units. In most systems, the mass, length, and time are considered to be fundamental quantities, and their units are called fundamental units. The following are some systems of units which are in common use.

- **c.g.s. system of units:** The unit of length is centimetre (cm). The unit of mass is gram (g). The unit of time is second (s)
- **m.k.s. system of units**: The unit of length is the metre (m). The unit of mass is the kilogram (kg). The unit of time is second (s)

- **f.p.s. system of units**: The unit of length is a foot (ft). The unit of mass is a pound (Lb). The unit of time is second (s). This system is no more in use.

S.I. System of Units:

In the year 1960, the Eleventh General Conference of Weights and Measures introduced the International System of Units. The International Standard Organization (ISO) and the International Electrochemical Commission endorsed the system in 1962. In October 1971 a replacement of the metric system of units was done with a new system called Systeme Internationale d' Unites.

Fundamental Units:

	Fundamental Quantity	S.I. Unit
1	Length	Metre
2	Mass	Kilogram
3	Time	Second
4	Temperature	Kelvin
5	Electric current	Ampere

6	Luminous intensity	Candela
7	Amount of substance	mple

Besides these seven basic units, there are two supplementary units. S.I. unit for the plane angle is radian (rad) and that of solid angle is steradian (sd).

Supplementary Units:

	Quantity	S.I. Unit
1	Plane angle	radian
2	Solid angle	steradian

This system of units is an improvement and extension of the traditional metric system. Now, this system of units has replaced all other systems of units in all branches of science, engineering, industry, and technology.

Guidelines for Writing SI Units and Their Symbols:

- All units and their symbols should be written in small case letters e.g. centimetres (cm), metre (m), kilogram per metre cube (kg m^{-3}).
- The units named after scientists are not written with a capital initial letter but its symbol is written in capital letter. Thus the unit of force is written as 'newton' or' N' and not as 'Newton'. Similarly unit of work and energy is joule (J), S.I. unit of electric current is ampere (A). The S.I. nit of pressure is pascal (Pa) and that of temperature is kelvin (K).
- No full stop should be placed after the symbol.
- The denominators in a compound unit should be written with negative powers. Thus an index notation should be used to write a derived unit. for example unit of velocity should be written as ms^{-1} instead of m/s. The unit of density is kilogram per metre cube (kg m^{-3} and not kg/m^3)
- No plural form of a unit or its symbol should be used. example 5 newtons should be written as 5 N and not as 5 Ns.
- A compound unit obtained from units of two or more physical quantities is written either by putting a dot or leaving a space between symbols of two

units. Example unit of torque is newton metre is written as Nm ot N.m. Unit of impulse is newton second is written as N s or N.s.

- Some space should be maintained between the number and its unit.

Advantages of S.I. System of Units:

- Units are simple to express
- This system uses only one unit for one physical quantity. Hence it is a rational system of units.
- Units of many physical quantities are related to each other through simple and elementary relationships For example 1 ampere = 1 volt / 1 ohm.
- It is a metric system of units. There is a decimal relationship between the units of the same quantity and hence it is possible to express any small or large quantity as a power of 10. i.e. inter-conversion is very easy. For e.g. 1kg = 1000 gm = 10^3 gm
- The physical quantities can be expressed in terms of suitable prefixes.
- a joule is a unit of all forms of energy and it is a unit of work. Hence it forms a link between mechanical and electrical units. Hence S.I. the system is a rational system because it uses only one unit for one physical quantity.
- This system forms a logical and interconnected framework for all measurements in science, technology, and commerce.
- All derived units can be obtained by dividing and multiplying the basic and supplementary units and no numerical factors are introduced as in another system of units. Hence S.I. system of units is a coherent system. Hence S.I. system of units is used worldwide.

General Steps to Find Derived Unit:

- Step -1 Write the formula for the quantity whose unit is to be derived.
- Step -2 Substitute units of all the quantities in one system of units in their fundamental or standard form.
- Step -3 Simplify and obtain derive unit of the quantity.

Example: To find the unit of velocity.

Velocity is a derived quantity. Hence its unit is a derived unit.

The velocity is given by, velocity = displacement/time

S.I. unit of velocity = S.I. unit of displacement/ S.I. unit of time = m/s

Thus S.I. unit of velocity is m/s

Definitions of Fundamental Units in S. I. System:

1 metre:

- The unit of length is a metre. Its symbol is 'm'.
- The distance travelled by electromagnetic waves in the vacuum in 1/299, 792, 458 seconds is called 1 metre. The denominator is the velocity of light in the vacuum which is in m/s and is known accurately.
- One metre is 1,650, 763.73 times the wavelength of orange light emitte by a krypton atom at normal pressure. The wavelength of light is precisely defined in terms of electron transition in an atom, is easily reproducible and is not affected by the change in place, time, temperature and pressure, etc. Hence metre is defined in terms of wavelength of orange light.

1 kilogram:

- The unit of mass is a kilogram. Its symbol is 'kg'.
- 1 kilogram is defined is the total mass of 5.0188 × 1025 atoms of C12 isotopes of carbon. Or The mass of a cylinder made up of platinum-iridium alloy kept at the International Beuro of Weights and Measure is defined as 1 kilogram. Reason for Using Platinum iridium alloy for the cylinder is that it is least affected by environment and time.

1 second:

- The unit of time is second. Its symbol is 's'.
- 1 second is a time duration of 9,192,631,770 periods of the radiation corresponding to the transition between two hyperfine levels of the ground state of the Cesium-133 atom. Period of vibration of the atom of Cesium – 133 is used for defining the standard of time because the period of vibration of the atom of Cesium – 133 are precisely defined, is easily reproducible and is not affected by a change in place, time, temperature and pressure, etc.

1-degree kelvin:

- The unit of temperature is degree kelvin. Its symbol is 'K'.

- 1-degree kelvin is a fraction 1/ 273.16 of the thermodynamic temperature of the triple point of the water. The triple point of the water is a temperature at which ice, water, and water vapour are in equilibrium.

1 candela:

- The unit of luminous intensity is candela. Its symbol is ‘cd’.
- 1 candela is luminous intensity in the normal direction of a surface of area 1/600000 m^2 of a black body at the freezing point of platinum under pressure of 1.01325×10^5 N/m^2.

1 ampere:

- The unit of electric current is the ampere. Its symbol is ‘A’.
- 1 ampere is the constant current, which is maintained in each of two infinitely long straight parallel conductors of a negligible cross-section, situated one metre apart in vacuum, will produce between the conductors a force of 2×10^{-7} N/m.

1 mole:

- The unit of the amount of substance is mole. Its symbol is ‘mol’
- 1 mole is the amount of substance which contains as many elementary entities (atoms, molecules, ions, electrons, etc.) as there are atoms in 0.012 kg of pure C12. The number of entities in one mole is $6.02252 \text{ X } 10^{23}$. It is called as Avagadro’s number.

1 radian:

- The unit of plane angle is the radian. Its symbol is ‘rad’
- One radian is defined as the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle.

1 steradian:

- The unit of solid angle is steradian. Its symbol is ‘sr’
- One steradian is defined as the solid angle that encloses a surface on the sphere of an area equal to the square of its radius.

Maintaining Uniformity of Standards

An international body Conference Generale des Poids et Measures or CGPM (General Conference of Weight and Measures) has been given the authority to decide the standards and units by international agreement. It holds its meetings and any change in the standard units are communicated through the publications of the Conference.

India adopted the metric system of units in 1956 by Parliament Act "Weights and Measures Act- 1956". The function of manufacturing, maintaining, monitoring, and improving the standards of measurements is discharged by the National Physical Laboratory (NPL), New Delhi. The uniformity in standards is maintained as follows:

- Measures (e.g. balances and weights) used by shopkeepers are expected to be certified by the Department of Measures and Weights of the local government.
- The working standards of these local departments have to be calibrated against the state-level standards, or any laboratory which is entitled to do so.
- The state-level laboratories are required to get their standards calibrated from the National Physical Laboratory at the national level, which is equivalent to international standards. Thus, measurements made at any place in the world are connected with the international system.

Prefixes Used in SI System:

Prefiks	Symbol	Multiplying factor
yotta	Y	1 000 000 000 000 000 000 000 000 = 10^{24}
zetta	Z	1 000 000 000 000 000 000 000 = 10^{21}
exa	E	1 000 000 000 000 000 000 = 10^{18}
peta	P	1 000 000 000 000 000 = 10^{15}
tera	T	1 000 000 000 000 = 10^{12}
giga	G	1 000 000 000 = 10^{9}
mega	M	1 000 000 = 10^{6}
kilo	k	1 000 = 10^{3}
hecto	h	100 = 10^{2}
deka	da	10 = 10^{1}
deci	d	0,1 = 10^{-1}
centi	c	0,01 = 10^{-2}
milli	m	0,001 = 10^{-3}
mikro	µ	0,000 001 = 10^{-6}
nano	n	0,000 000 001 = 10^{-9}
piko	p	0,000 000 000 001 = 10^{-12}
femto	f	0,000 000 000 000 001 = 10^{-15}
atto	a	0,000 000 000 000 000 001 = 10^{-18}
zepto	z	0,000 000 000 000 000 000 001 = 10^{-21}
yocto	y	0,000 000 000 000 000 000 000 001 = 10^{-24}

Examples to Understand the Use of Units in Numerical Problems

Use of standard prefixes used in S.I. system to express the following quantities:

- 10^6 phones (1 Mphones)
- 10^{-6} (1 μphones)
- 10^{12} (1 Tcows)
- 10^{-9} monkeys(1 nmonkeys)
- 10^{-12} birds (1 pbirds)
- 12×10^{-9} dogs (12 ndogs)
- 34×10^3 boys (34 kboys)Numerical Problems